Modelling the Off-Shell Dependence of $\pi^o - \eta$ Mixing with Quark Loops

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ABSTRACT

It is shown that including form factors for the quark-pseudoscalar meson couplings of the Georgi-Manohar chiral quark model allows one to obtain the leading off-shell dependence of $\pi^o - \eta$ mixing (as predicted by chiral perturbation theory) from the effect of quark loops on the meson propagators. Implications for $\rho^o - \omega$ mixing and for the effects on meson mixing contributions to few body charge symmetry breaking observables are also discussed.

INTRODUCTION

It is commonly held that the bulk of non-electromagnetic charge symmetry breaking (CSB) in few-nucleon systems is due to isoscalar-isovector mixing in the intermediate meson propagators of one-boson-exchange graphs. The dominant contributions of this type are associated with $\rho^o - \omega$ and, to a lesser extent, $\pi^o - \eta$ mixing [1-4]. Recently, however, it has been pointed out that some problems exist with the standard evaluations of these contributions.

First, recall that the $\rho^o - \omega$ mixing matrix element is obtained from experimental data on $e^+e^- \to \pi^+\pi^-$ in the $\rho - \omega$ interference region and therefore corresponds to $q^2 \simeq m_\omega^2$. The standard analysis assumes this matrix element is q^2-independent and, therefore, uses the experimental value unchanged in the NN scattering region, for which $q^2 < 0$. Not only has the validity of this ansatz never been tested, but, in Ref. 5, using a model in which $\rho^o - \omega$ mixing is generated by an intermediate quark loop as a consequence of the inequality of up and down quark masses, Goldman, Henderson, and Thomas (GHT) raise the possibility of significant q^2-dependence of the $\rho^o - \omega$ mixing matrix element.

Second, a recent evaluation [6] of $\pi^o - \eta$ mixing to one loop in chiral perturbation theory (ChPT), shows that 1) the mixing matrix element is, indeed, q^2-dependent (varying by $\sim$ 20% over a range from $q^2 \simeq m_\eta^2$ to $q^2 \simeq -m_\eta^2$, comparable to that involved in the extrapolation of the $\rho^o - \omega$ matrix element from $q^2 \simeq m_\omega^2$ to the scattering region) and 2) the magnitude of the mixing, even without this q^2-dependence, is less than obtained by the model analysis of Ref. 3c. The results of Ref. 6, being at one loop in ChPT, provide only the linear-in-q^2 dependence of $\pi^o - \eta$ mixing; the higher order dependence is unconstrained but will be small in the region of validity of the chiral expansion ($|q^2| \lesssim m_\eta^2$). Since the convergence of the chiral expansion to one loop in this region has been extensively tested [7,8], and the framework of ChPT is a rigorous consequence of QCD, the results of Ref. 6 should be considered very reliable (up to an overall $\sim$20% scale uncertainty associated with the treatment of electromagnetic corrections of pseudoscalar masses only to leading order in the chiral

expansion [6,9]).

The aim of the present paper is to use the constraints of Ref. 6 to test the type of model building which underlies the GHT approach. As we will see, the modelling does, indeed, succeed in reproducing the correct q^2-dependence of $\pi^o - \eta$ mixing, which considerably strengthens the case of GHT that, as a consequence of the q^2-dependence of the mixing, the dominant $\rho^o - \omega$ mixing contribution to few-body CSB may be significantly different than previously thought.

THE Q^2-DEPENDENCE of $\pi^o - \eta$ MIXING AND THE QUARK LOOP MODEL

Let us begin with a few general remarks. These are necessitated by the, apparently widely held, view that mixing matrix elements of the sort we are discussing are expected to be "naturally" q^2-independent. This is simply not the case.

In fact the whole assumption underlying the meson exchange framework is that, at low energies, QCD reduces to an effective low energy theory involving only composite hadronic fields (meson, nucleons, deltas...). Let us assume, for the sake of the argument in this paper, that this assumption is essentially correct. Then, whatever the effective theory governing the interactions of these hadrons <u>is</u>, we know that, as an effective low energy theory, it will be described by a non-renormalizable Lagrangian in which all terms not explicitly forbidden by the symmetries of the underlying theory (QCD) occur. In particular, there will be terms in the meson sector of the theory which involve the quark mass matrix and higher powers of derivatives, which will naturally lead to q^2-dependent meson mixing. The pseudoscalar sector, where we actually know something about the effective theory beyond leading order in the momentum expansion, is one explicit example of this general principle. The standard <u>ansatz</u>, of taking, eg., the $\rho^o - \omega$ matrix element to be independent of q^2, is thus incompatible with the assumptions underlying the framework in which such mixing is to be used to generate CSB in few-body systems. Of course, without knowing the terms of effective Lagrangian involving the vector mesons beyond leading order, one does not know what the magnitude of the q^2-dependence is, and hence whether the

effect of the q^2-running is important or not. Given the pseudoscalar result and the natural (QCD) scales involved, however, it is unlikely that the effect will be negligible. The fact that the vector mesons are much less point-like than the pseudscalar mesons further supports this contention. [1]

Let us now turn to the question at hand, namely, whether or not a quark-loop model of the GHT type is capable of reproducing the behavior of $\pi^o - \eta$ mixing known from ChPT. To one-loop in ChPT, Ref. 6 shows that one obtains a q^2-dependent $\pi_3 - \pi_8$ mixing angle given by

$$\theta(q^2) = \frac{\sqrt{3}(m_{K^o}^2 - m_{K^+}^2)_{QCD}}{(m_K^2 - m_\pi^2)}$$

$$\left[1 + \Delta_{GMO} + \frac{1}{16\pi^2 f^2}\left(\frac{m_\eta^2}{m_\eta^2 - m_\pi^2}\right)\right.$$

$$\times \left(3m_\eta^2 \ln(m_K^2/m_\eta^2) + m_\pi^2 \ln(m_K^2/m_\pi^2)\right)$$

$$\left. + \left(\frac{q^2 + m_\eta^2}{32\pi^2 f^2}\right)\left(1 + \left(\frac{m_\pi^2}{m_K^2 - m_\pi^2}\right)\ln(m_\pi^2/m_K^2)\right)\right] \tag{1}$$

where π_3, π_8 are the unmixed states to which the physical π^o, η states reduce in the limit of isospin symmetry, Δ_{GMO} is the Gell-Mann-Okubo discrepancy

$$\Delta_{GMO} = (4m_K^2 - m_\pi^2 - 3m_\eta^2)/(m_\eta^2 - m_\pi^2), \tag{2}$$

[1]Note also that the non-renormalizable structure of the effective low energy Lagrangian will lead to a q^2-dependence of the “renormalized” (in the sense of the quark mass/momentum expansion) self-energies for the composite fields. This means that the meson propagators in the effective theory may differ significantly from the form

$$\Delta(q) = i/(q^2 - m^2)$$

away from the pole (i. e. especially in the spacelike region relevant to NN scattering in meson exchange models). The pseudoscalar mesons are special in this regard since, as one can readily see from the structure the effective Lagrangian [see e. g. Ref. 7], the q^2-dependence of the renormalized self-energy enters first at two-loop order and hence will be very small, by the usual power counting arguments, in the region of validity of the chiral expansion. This is a special property associated with the chiral constraints of the pseudoscalar sector and will not be a property, e. g. of vector meson fields.

f is a parameter of the chiral expansion (equal to f_π in leading order) and $(m^2_{K^o} - m^2_{K^+})_{QCD}$ is the contribution to the kaon mass splitting due to $m_d \neq m_u$, obtained by correcting the observed splitting for electromagnetic effects. This correction is usually performed using only leading order results in the electromagnetic chiral expansion (Dashen's theorem [10]). If this procedure were accurate, the corrections to (1), which enter only at 6th order in the low-energy expansion, would be small in the region of small q^2, say $|q^2| \lesssim m^2_\eta$.

An old attempt by Socolow [11] to saturate the Cottingham formula for the kaons with K, K* states, however, indicates that there may be significant corrections to Dashen's theorem. This possibility is also discussed in Ref. 9, on the basis of the size of the relevant electromagnetic chiral logarithms. At present, both the kaon electromagnetic splitting obtained from Dashen's theorem and the larger Socolow value are compatible with other isospin breaking data [9]. The larger Socolow value would lead to a value of $(m^2_{K^o} - m^2_{K^+})_{QCD}$ 30% <u>larger</u> than that obtained using Dashen's theorem. There is thus an overall scale uncertainty in (1) (which uncertainty, however, can be reduced by improved experimental results,
e. g. on isospin breaking in K_{e3}).

Our aim now is to see whether a quark-loop model of the GHT type succeeds in reproducing the constraints just discussed. Two assumptions underlie the GHT approach: first, that the behavior of the full low-energy meson theory can be obtained by integrating out quarks from a theory consisting of free mesons coupled to quarks and, second, that the effect of the quark loops can be adequately represented by keeping only those loops generated by the lowest order quark-meson couplings of the effective quark-meson theory using a) a monopole form factor at each quark-meson vertex, and b) free constituent quark propagators. The basic philosophy behind these assumptions is that the higher order terms in the effective meson Lagrangian arise from the non-pointlike, quark substructure of the mesons, and that incorporating this substructure in a way that (through the monopole form factor parameter, Λ) is capable of reflecting the actual meson size should allow one to reproduce the behavior associated with these terms.

For the problem at hand we, therefore, start with a model which incorporates both quarks and pseudoscalar mesons. The model must, moreover, properly respect the approximate chiral symmetry with which the pseudoscalars are associated as pseudo-Goldstone bosons. A natural choice is the Georgi-Manohar chiral quark model [12,13]. The model describes an effective theory of massive constituent u, d, s quarks and the pseudoscalar octet, $\{\pi^a\}$, transforming non-linearly under chiral $SU(3)_L \times SU(3)_R$ according to

$$\xi \rightarrow \xi' = L\xi U^+ = U\xi R^+ \tag{3a}$$

$$q \rightarrow q' = Uq \tag{3b}$$

where $\xi = exp(i\lambda \cdot \pi/2f)$ is the square root of the matrix $\Sigma = exp(i\lambda \cdot \pi/f)$, $\{ \lambda^a \}$ are the usual Gell-Mann matrices and f is as in Eqn. (1) above. Σ transforms linearly under $SU(3)_L \times SU(3)_R$,

$$\Sigma \rightarrow \Sigma' = L\,\Sigma\,R^+. \tag{4}$$

In (3) and (4), U reduces to the usual flavor $SU(3)$ matrix for an $SU(3)_V$ transformation and is a non-linear function of L, R, $\{\pi^a\}$ otherwise. In the chiral limit (in which the model is to be invariant under the transformations (3a),(3b)), if we consider only terms with zero or one derivative (the leading terms in the momentum expansion), the effective Lagrangian of the model is

$$Ł^{(o)}_{eff} = -m\bar{q}q + i\bar{q}\,\not{D}\,q + g_A\,\bar{q}\,\not{\mathbf{A}}\gamma_5\,q \tag{5}$$

where the covariant derivative, D_μ, is given by

$$D_\mu = \partial_\mu - i\mathbf{V}_\mu \tag{6}$$

and the vector and axial vector fields $\mathbf{V}_\mu$, $\mathbf{A}_\mu$ are defined by

$$\mathbf{V}_\mu = \frac{1}{2}(\xi^+\partial_\mu\xi + \xi\partial_\mu\xi^+)$$

$$\mathbf{A}_\mu = \frac{i}{2}(\xi^+\partial_\mu\xi - \xi\partial_\mu\xi^+) \tag{7}$$

In Eqn. (5), g_A is the constituent quark axial coupling, $g_A \simeq 0.75$, and m is the hypothetical constituent quark mass, in the chiral limit. At this stage chiral symmetry is unbroken, the π^a are all massless, isospin is exact, and there is no $\pi_3 - \pi_8$ mixing. Chiral symmetry breaking is then incorporated using the standard techniques of effective chiral Lagrangians. The leading symmetry breaking term involving both quarks and pseudoscalars is

$$Ł_B = \frac{-Cm}{\Lambda_\chi^2}\bar{q}(\xi\mu M\xi + \xi^+\mu M\xi^+)q \tag{8}$$

where μ is a mass scale related to the quark condensate, M is the current quark mass matrix, $M = diag(m_u^c, m_d^c, m_s^c)$, Λ_χ is a chiral symmetry breaking scale $\sim$ 1GeV and C is expected to be of order 1. C is in fact constrained by the observation that the terms zeroth order in π^a in Eqn. (8) produce a splitting of the constituent s mass from the constituent u, d masses

$$\delta m_s^{con} \equiv m_s^{con} - m_{u,d}^{con} \simeq \frac{2Cmm_K^2}{\Lambda_\chi^2} \sim 200MeV \tag{9}$$

where the lowest order mass relations for the pseudoscalars have been used to set $\mu m_s \simeq m_K^2$. For further details of the model, the reader is referred to Refs. 12,13.

According to the GHT ansatz, what we are now supposed to do is 1) drop all additional terms in the chiral expansion for the quark, pseudoscalar, and quark-pseudoscalar sectors, and 2) hope to generate the effects of these terms by including a monopole form factor, $F(k^2) = \Lambda^2/(\Lambda^2 - k^2)$, for each meson leg coupling to a quark line (with k the four-momentum flowing through the vertex on the quark line) and considering the effect of quark loops with free, constituent quark propagators.

Such loops contribute to the pseudoscalar propagators in two ways. First, the axial vector coupling term in Eqn. (5) generates pseudovector quark-pseudoscalar meson couplings, which in turn generate two-vertex loops as in Fig. 1. (Because of the pseudovector nature of the couplings, these loops are proportional to q^2 near $q^2 = 0$ and do not contribute to the pseudoscalar masses.) Second, the terms of $Ł_B$ second order in the pseudoscalar fields produce tadpole diagrams, as in Fig. 2. These loop contributions are proportional to <u>current</u> quark masses and naturally lead to

the usual relations between the pseudoscalar squared-masses and the current quark masses,

$$
\begin{aligned}
m_\pi^2 &= \mu(m_u^c + m_d^c) \\
m_{K^+}^2 &= \mu(m_s^c + m_u^c) \\
m_{K^o}^2 &= \mu(m_s^c + m_d^c) \\
m_\eta^2 &= \mu(\frac{4}{3}m_s^c + \frac{1}{3}m_u^c + \frac{1}{3}m_d^c)
\end{aligned}
\tag{10}
$$

up to an overall factor which is Λ-dependent, as we will see in more detail below.

Requiring that we reproduce the correct leading mass relations would, therefore, fix the value of Λ to be used in the GHT ansatz. Choosing Λ in this way then automatically also fixes the $\pi_3 - \pi_8$ mixing angle to its correct, leading order value

$$\theta^{leading} = \frac{\mu(m_d - m_u)}{\sqrt{3}(m_\eta^2 - m_\pi^2)}. \tag{11}$$

The real test of the GHT ansatz is then whether or not it is able to reproduce the correct q^2-dependence of θ, as given in Eqn. (1).

To evaluate $\theta(q^2)$ in the model, we include the quark loops of Figs. 1,2. The $\pi_3 - \pi_8$ inverse propagator then takes the form

$$\mathbf{\Delta}^{-1}(q^2) = \begin{bmatrix} q^2 - \pi_{33}(q^2) & -\pi_{38}(q^2) \\ -\pi_{38}(q^2) & q^2 - \pi_{88}(q^2) \end{bmatrix}. \tag{12}$$

In Eqn. (11), $\pi_{33}, \pi_{38}, \pi_{88}$ all include both tadpole and two-vertex-loop contributions. The former are q^2-independent, the latter q^2-dependent (and beginning at $O(q^2)$ near $q^2 = 0$). π_{38}, moreover, is proportional to $(m_d^c - m_u^c)$. For small q^2 (the only values for which we have a constraint) we may write

$$\pi_{kl}(q^2) = \pi_{kl}^{(0)} + q^2\pi_{kl}^{(1)} \tag{13}$$

where the $\pi_{kl}^{(0)}$ are associated with the tadpoles and the $\pi_{kl}^{(1)}$ with the two-vertex loops. Then the angle which diagonalizes $\mathbf{\Delta}^{-1}$, to $O(m_d - m_u)$ (the same order as the ChPT result in Eqn. (1)), is given by

$$\theta(q^2) = \frac{-\pi_{38}(q^2)}{(\pi_{88}(q^2) - \pi_{33}(q^2))}$$

$$\simeq \frac{-\pi_{38}^{(0)}}{\pi_{88}^{(0)} - \pi_{33}^{(0)}} \left[1 + q^2 \left(\frac{\pi_{38}^{(1)}}{\pi_{38}^{(0)}} - \frac{(\pi_{88}^{(1)} - \pi_{33}^{(1)})}{(\pi_{88}^{(0)} - \pi_{33}^{(0)})} \right) \right] \tag{14}$$

where we have kept only terms up to $O(q^2)$ in the expansion occuring on the second line of Eqn. (14). The appearance of the leading order expression as an overall factor in Eqn. (14) ensures that the scale of $\theta(q^2)$ at $q^2 = 0$ is the leading order result, Eqn. (1), when Λ is chosen so as to give the leading order results for the pseudoscalar masses. There are, in fact, corrections to the leading order pseudoscalar mass expressions at one-loop in ChPT which would affect the choice of Λ, but these are not fixed numerically with great precision because of residual uncertainties in the higher order low-energy constants of Ref. 7. The resulting overall scale uncertainty in Eqn. (14) is not, however, a practical difficulty for three reasons. First, we know that the value should be close to that required to give the correct leading order mass formula and that this value of Λ gives the leading order result for θ, which will be close to the correct magnitude at $q^2 = 0$ even after one-loop corrections. Second, there is a range of uncertainty of 30% in the overall scale of the result Eqn. (1) as a consequence of the uncertainties in the electromagnetic corrections to the kaon splitting. And third, it turns out that the slope with q^2 of the factor in brackets in Eqn. (14) is rather weakly dependent on Λ. We will, therefore, quote all results below with Λ fixed so as to give the leading order pseudoscalar mass relations. The potential error in the slope with q^2 is $\sim 10\%$.

THE QUARK LOOP CONTRIBUTIONS

To generate the loop contributions of Figs. 1,2 we require the relevant quark-pseudoscalar meson couplings. The pseudovector couplings required for Fig. 1 come from the third term in Eqn. (5) and are given by

$$\frac{-g_A}{2f}\left(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right)\partial^\mu\pi_3 - \frac{g_A}{2\sqrt{3}f}\left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s\right)\partial^\mu\pi_8$$

$$\equiv \sum_{\substack{q=u,d,s\\ a=3,8}}\left(\frac{g_{PV}^{a,q}}{f}\right)\bar{q}\gamma_\mu\gamma_5 q\partial^\mu\pi^a \qquad (15)$$

where the second line defines the couplings, $g_{PV}^{a,q}$. The tadpoles of Fig. 2 are produced by terms which couple two pseudoscalars to a quark line. The contributions associated with the vector current part of the second term in Eqn. (5) vanish by Lorentz invariance. The surviving tadpole contributions are then generated by those pieces of $Ł_B$ (Eqn. (8)) second order in the $\{\pi^a\}$, the terms involving π_3, π_8 being

$$\frac{cm\mu}{\Lambda_\chi^2 f^2}\left[(m_u^c\bar{u}u + m_d^c\bar{d}d)(\pi_3)^2 + \frac{2}{\sqrt{3}}(m_u^c\bar{u}u - m_d^c\bar{d}d)\pi_3\pi_8\right.$$

$$\left.+\frac{1}{3}(4m_s^c\bar{s}s + m_u^c\bar{u}u + m_d^c\bar{d}d)(\pi_8)^2\right]$$

$$\equiv \sum_{\substack{q=u,d,s\\ ab=33,88,38}} g_{tad}^{ab,q} m_q^c\bar{q}q\pi^a\pi^b \qquad (16)$$

where the second line defines $g_{tad}^{ab,q}$.

From Eqn. (15) we find that the 2-vertex-loop contribution to the ab element of the mass matrix, $\pi_{loop}^{(ab)}(q^2)$, is given by

$$\pi_{loop}^{ab}(q^2) = \sum_{q=u,d,s}\left(\frac{-ig_{PV}^{a,q}g_{PV}^{b,q}}{2!\ f^2}q^\mu q^\nu\right)$$

$$\cdot\int\frac{d^4k}{(2\pi)^4}\frac{Tr[\gamma_\mu\gamma_5(\frac{1}{2}\not{q}+\not{k}+m_q)\gamma_\nu\gamma_5(-\frac{1}{2}\not{q}+\not{k}+m_q)]}{[(\frac{1}{2}q+k)^2-m_q^2][(-\frac{1}{2}q+k)^2-m_q^2]}\left(\frac{\Lambda^2}{(\Lambda^2-k^2)}\right)^2. \qquad (17)$$

This integral is readily evaluated using standard Feynman parameter techniques. The resulting expression is somewhat lengthy and will not be quoted here since we require only the leading, $O(q^2)$, contribution. This may be obtained straightforwardly from the full expression, and is given by

$$\pi_{ab}^{(1)} \equiv \frac{\partial \pi_{loop}^{(ab)}(q^2)}{\partial q^2}\Big|_{q^2=0} = \sum_{q=uds} \left(\frac{g_{PV}^{a,q} g_{PV}^{b,q}}{f^2} \right) \frac{\Lambda^4}{8\pi^2}$$

$$\left[\frac{(\Lambda^2 + 5m_q^2)}{2(\Lambda^2 - m_q^2)^2} - \frac{(2\Lambda^2 + m_q^2)m_q^2}{(\Lambda^2 - m_q^2)^3} ln\left(\Lambda^2/m_q^2\right) \right]. \tag{18}$$

Similarly, from Eqn. (16), we obtain the (q^2-independent) tadpole contribution to the mass matrix, π_{ab}

$$\pi_{ab}^{(0)} = \sum_{q=uds} g_{tad}^{ab,q}\ m_q^c(1+\delta_{ab}) \int \frac{d^4k}{(2\pi)^4} Tr\left[\frac{i}{\not{k} - m_q} \right] \left(\frac{\Lambda^2}{\Lambda^2 - k^2} \right)^2 \tag{19}$$

$$= (1+\delta_{ab}) \sum_{q=uds} g_{tad}^{ab,q}\ m_q^c\ Q\left(\Lambda, m\right)$$

where

$$Q(\Lambda, m) \equiv \frac{m\Lambda^4}{4\pi^2} \left[\frac{1}{(\Lambda^2 - m^2)} - \frac{m^2}{(\Lambda^2 - m^2)^2} ln\left(\Lambda^2/m^2\right) \right] \tag{20}$$

and, in the second line of Eqn. (19), we have retained only those terms linear in the current quark masses, m_q^c. Eqn. (19), together with Eqn. (9), implies

$$\pi_{33}^{(0)} = \left[\frac{\delta m_s^{con} Q(\Lambda, m)}{m_K^2 f^2} \right] \mu(m_u^c + m_d^c)$$

$$\pi_{88}^{(0)} = \left[\frac{\delta m_s^{con} Q(\Lambda, m)}{m_K^2 f^2} \right] \mu \left(\frac{4}{3} m_s^c + \frac{1}{3} m_u^c + \frac{1}{3} m_d^c \right) \tag{21}$$

$$\pi_{38}^{(0)} = - \left[\frac{\delta m_s^{con} Q(\Lambda, m)}{m_K^2 f^2} \right] \frac{\mu(m_d - m_u)}{\sqrt{3}}$$

which, as promised, are the correct leading order results providing Λ is chosen so that

$$Q(\Lambda, m) = m_K^2 f^2/\delta m_s^{con}. \tag{22}$$

RESULTS

In order to employ the results of the last section in Eqn. (14), we require a value for the constituent quark mass, m. An extensive analysis of the meson [14] and baryon [15] sectors, which includes kinematic relativistic corrections, obtains $m = 220MeV$, $\delta m_s^{con} = 200MeV$. As an alternate value for m, and in order to display the sensitivity to the parameters, we will also quote results for $m = \frac{3}{2}m_{\pi^o}$, which would put the threshold for the mixing matrix element in the correct location. Rewriting Eqn. (1) as

$$\theta^{ChPT}(q^2) = \theta(q^2 = 0)[1 + c_1 q^2] \tag{23}$$

we have, from Eqn. (1), that $c_1 = 0.279GeV^{-2}$.

In Table 1 we present the model results for c_1. The errors quoted correspond to allowing Λ to vary over a range for which the coefficients of the leading order results in Eqn. (21) vary by $\pm 20\%$ about the value 1 (a typical variation associated with one-loop corrections in ChPT). As may be seen from the Table, this error amounts to $\sim 10\%$. c_1 also varies by $\sim 5\%$ between $m = \frac{3}{2}m_{\pi^o}$ and $m = 220MeV$. A similar variation is produced by varying δm_s^{con} by $\sim 10\%$. As we see from the Table, the agreement with the ChPT result is quite good. This gives us confidence that the GHT type of modelling may give reliable results for the off-shell dependence of mixing, at the very least to lowest non-trivial order in q^2.

It should be pointed out, in passing, that the GHT calculation only investigated the q^2-dependence of the off-diagonal $\rho - \omega$ matrix element, $\pi_{\rho\omega}$, in the inverse propagator, analogous to π_{38} of Eqn. (11). When one, however, extracts the $\rho - \omega$ element of the propagator itself and writes it in the form

$$\frac{i\pi_{\rho\omega}^{phen}(q^2)}{(p^2 - m_\rho^2)(p^2 - m_\omega^2)}, \tag{24}$$

the q^2-dependence of $\pi_{\rho\omega}^{phen}(q^2)$ arises not only from the proportionality to $\pi_{\rho\omega}$, but also from the q^2-dependence of the ρ, ω renormalized self-energies. The actual GHT predictions correspond to $\pi_{\rho\omega}$ and <u>not</u> $\pi_{\rho\omega}^{phen}$ and so should not be employed, in their present form, in few-body CSB calculations.

The present calculation, however, suggests that the full version of the GHT calculation for $\rho\omega$ mixing should provide a reliable estimate of the off-shell behavior of the $\rho\omega$ matrix element, at least in the region of small q^2. If one, further, wished to take the success of the calculation as evidence for the validity of the physical argument underlying the GHT ansatz, one might hope that the higher order q^2-dependence would also be well-modelled. We have, of course, no way of demonstrating that this will be the case; however, from both the present results, those of ChPT, and from general principles, it is clear that a model which passes the test of satisfying at least one known constraint should be considered more reliable than one (the standard ansatz of a q^2-independent matrix element) which fails it. This means that the few-body CSB contributions to such observables as the difference of n and p scattering lengths and the non-Coulombic $A = 3$ binding energy difference need re-evaluation. This can only be done reliably (i. e. in the regime where the modelling has been tested) to the extent that meson-mixing contributions are associated primarily with the region of q^2 for which the leading q^2-dependence of the mixing matrix element is dominant.

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TABLE 1. The slope of $\theta(q^2)$ with q^2 as a function of $m, \delta m_s^{con}$

m(MeV)	δm_s^{con}(MeV)	$c_1(GeV^{-2})$
202.5	175	$.223 \pm .026$
	200	$.234 \pm .026$
	225	$.244 \pm .029$
220	175	$.211 \pm .024$
	200	$.222 \pm .025$
	225	$.231 \pm .026$

Figure Captions

1. Pseudovector-coupling-induced contributions to the meson propagator matrix.

2. Tadpole contributions to the meson propagator matrix.